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The Construction of the Optimal Investment Portfolio Using the Kelly Criterion

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In general, the purpose of investment is to amass a more wealth in the future on the basis of a current investment capital. If any investment alternative will certainly generate profit in the future, we will invest all capital in the alternative. But in reality, all future values regarding to investing activities are uncertain. Therefore to achieve a goal of the investment, we need to minimize uncertainty inherent in the investment by constructing an optimal portfolio. Namely, it means we optimize an allocation rate of investment capital for the diversified investment. In this paper, we will discuss the Kelly criterion which is one of the methods to determine the optimal allocation of the investment capital resulting in maximizing a geometric expected value of the return over a long period of time. Then, we will construct the optimal portfolio with data of the KOSPI 200 using the Kelly criterion as a real case study.

JEL Codes: C51, G11 and G14

1. Introduction

People have been greatly interested in investing as their wealth increases. As a result, the books guiding their investment have been rapidly explored, especially related to the stock investment. In these days, the stock investors are concerned about investing strategies instead of simply investing in stocks as they had done before. By so doing, some of the individual investors may accomplish a surprisingly higher rate of return on the stock investment than a bench mark indices (Poundstone 2006). What the most important thing should be done in a stock investment is to predict a behavior of stock prices in an uncertain market environment accurately as much as possible even if it is not easy job to be done.

When investing in stocks, all investors make a great deal of their effort so as to fulfill a risk-adjusted rate of return on a stock investment as we want to obtain a high expected rate of return on gambling games and horse-racing which are much dependent on a betting strategy. To the end, the investors need to construct an optimally diversified investment portfolio and invest in it, which is alternative to investing a full amount of investment capital in one stock solely. It implies that an investment capital must be optimally allocated to each of the stocks included in the stock portfolio to come by the high rate of return. The most general way to do the work is to use a Markowitz's mean-variance portfolio theory. General mean-variance theory does not address compounding of investment. But realistic theory with regard to investment mostly cannot help dealing with reinvestment because accumulated wealth according to compounding concerning investment is

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unimaginable higher than other cases. So to maximize the compound return in the long term we must construct the portfolio with maximizing geometric average rather than arithmetical average. It is the purpose of *Kelly Criterion*.

This study focuses on a real case study about the Kelly criterion. In section 2, we explain why we conduct a study about the optimal investment portfolio using the Kelly criterion through literature review. And in section 3, we describe what is the Kelly criterion, then we show how to apply for it in the stock market in section 4. Next, in section 5 we account for the empirical analysis of the Kelly criterion applied for a stock investment listed in KOSPI 200. Finally, we describe the results of this study together with critical issues in section 6.

2. Literature Review

The Kelly criterion has been extensively studied by many researchers since Kelly published the paper containing it in 1956 (Rothstein 1972, Rotando & Thorp 1992, Thorp 1997, Poundstone 2006). Bellman and Kalaba (1957) generalized and expanded the Kelly criterion incorporating a dynamic programming into the Kelly criterion. And Brieman (1961) proved that the Kelly criterion asymptotically minimized time taken to accomplish a certain level of wealth and maximized wealth within the least period of a time horizon. Thorp (1971, 1997) maintained that the Kelly criterion should take a place of a Markowitz's portfolio theory and applied it for the portfolio selection. Finkelstein and Whitley (1981) extended the Kelly and Brieman's results and demonstrated that the investors who made investment with the Kelly criterion to gain a superior rate of return that those who did without it on average. As the aforementioned researchers proposed, Ziemba and Hausch (1985) recognized that the Kelly criterion generated pretty much higher expected capital growth than any other investment strategies when investment was made over a long period of time. In addition to the work above, Ralph Vince (1990) popularized the Kelly criterion with a terminology of 'optimal f'. Browne and Whitt (1996) developed the Kelly Bayesian version of an optimal gambling and investment policies for cases which the underlying stochastic process had parameter values that were unobserved random variables. All the aforementioned researchers discussed the Kelly criterion focusing on gambling games and traditional stock investment, but Aurell, et. al (2000) applied the Kelly criterion for determining prices of derivative financial instruments in an incomplete market.

As a practical evidence of the superiority of the Kelly criterion over other investment betting strategies, Ralph Vince (1990) reported in his book that Shannon had gained an annual average rate of return of 28% in a stock investment over 35 years with the Kelly criterion, and Thorp and Ziemba also came by greater rate of return than S&P 500 in gambling games and stock investment applying the Kelly criterion for them. As a strong proponent of the Kelly criterion, Thorp established an investment company in 1969 and obtained an annual average rate of return of 15% from 1969 to 1980 when he closed his company due to a kind of an insider trading crime committed by his subordinate. Comparing Thorp's accomplishment with 88% of the return of S&P 500, his was judged to be greater than twice as much as the return of S&P 500.

According to Thorp's analysis, it was believed to be plausible that the Kelly criterion had been greatly contributed to Warren Buffet's amazing rate of return on a stock investment for a long period of time (Thorp 1990).

In summary, it can be said that the Kelly criterion has played a critical role in which investors achieve a high rate of return on investing and gambling games. Unfortunately, little research has been done on the Kelly criterion in Korea, though. So we will apply the Kelly criterion for the stocks listed in KOSPI to construct an optimal portfolio and to see what will happen to the KOSPI market with it.

3. What is the Kelly Criterion?

In this section, we will introduce the basic concept of the Kelly criterion (Kelly 1956, Thorp 1971, Thorp 1997, Ziemba & Ziemba 2007).

3.1 The Assumptions behind the Kelly Criterion

The main purpose of the Kelly criterion is to allow investors to determine the optimal betting ratio of investment capital to maximize the expected logarithmic rate of return on investment. By doing so, the investors may accumulate a maximum amount of his/her wealth. To implement the Kelly criterion, there are several basic assumptions as follows:

- The expected rate of return must be greater than 0.
- It must be possible to reinvest the cash flows earned from investment.
- There must exist an independent relationship among the final cash flows.
- The cash flows must follow a Bernoulli process.
- The investing activities must be performed over a long period of time.
- It must be possible to divide an investment capital infinitesimally.

When carrying out the Kelly criterion with the assumptions mentioned above, the investors may obtain a geometrically expected rate of return on investment.

3.2 The Generalization of the Kelly Criterion

The general mathematical formula of the Kelly criterion is expressed as Equation 1. In the equation, "G" represents a geometrical growth of a rate of return, "N" does a number of periods of investing time, and "X_0" does an initial amount of investment capital.

$$G = \lim_{N \to \infty} (1/N) \ln(X_N / X_0) \quad (1)$$

Let's derive a simple form of the Kelly criterion using Equation (1). For example, suppose that A and B play a game of flipping a coin. If a coin falls faceup, A wins a game. Otherwise, B wins the game. For the purpose of the current discussion regarding to the Kelly criterion, we need to assume that a probability to have a face up of a coin is a little bit greater than 1/2. In this game, if a player wins a game, he/she will receive a certain amount of return. Otherwise, he/she will lose a whole amount of money to bet. Let's consider the game in terms of a player "A" with an odd system of "1 to 1". Then, if the coin falls faceup in the first game, A will receive what he/she bet of "f", which is equivalent to a fraction of the investment capital to bet. Then after the first game, A's investment capital increase by $X_0 \times f$ and is expressed in Equation (2). If A loses the game, A's investment capital decreases by $X_0 \times f$ and is given by Equation (3).

$$X_1 = X_0 + X_0 f = X_0(1+f) \tag{2}$$

$$X_1 = X_0 - X_0 f = X_0(1-f) \tag{3}$$

For a simple discussion, let suppose that A won the first game and lost the second game, then A's investment capital after the second game is expressed by Equation (4).

$$X_2 = X_0(1+f) - X_0(1+f)f = X_0(1+f)(1-f) \tag{4}$$

In a similar manner, if A wins "W" times and loses "L" times out of "N" games, then A's accumulated investment capital after "N" games is equivalent to Equation (5).

$$X_N = X_0(1+f)^W(1-f)^L \tag{5}$$

Dividing both sides of Equation (5) by X_0 and taking a logarithm to the equation, then a logarithmic rate of return in investment is expressed by Equation (6).

$$G(f) = \ln(X_N / X_0) = W\ln(1+f) + L\ln(1-f) \tag{6}$$

When playing a game over a long period of time, then Equation (6) becomes Equation (7).

$$g(f) = p\ln(1+f) + q\ln(1-f) \tag{7}$$

Now we may find out the value of "f" which maximizes $g(f)$ in Equation (7). It is an optimal fraction of a betting ratio and denoted by "f^*". Then its expression is given by Equation (8).

$$\frac{dg(f)}{df} = \frac{p}{1+f} - \frac{q}{1-f} = 0$$

$$f^* = p - q = 2p - 1 \tag{8}$$

The term of "f^*" is called a Kelly criterion or Kelly system. Replacing Equation (8) into Equation (7), then Equation (7) is simplified in Equation (9).

$$g(f^*) = p\ln p + q\ln q + \ln 2 \tag{9}$$

Equation (9) is a maximum rate of return or growth rate of investment capital.

4. The Kelly Criterion Applied for a Stock Investment

The Kelly criterion is a mathematical model to effectively diversify an investment capital based on a geometrical expectation rule. When implementing the Kelly criterion, care should be exercised that the Kelly criterion relies on an odd system. Let's employ the most popular form of the Kelly criterion which is generally expressed as

$f^* = \frac{\text{expectation per unit bet}}{\text{gain per unit bet}} = \frac{edge}{odd}$.

Here, "edge" means an expected rate of return when investment is made with a same probability and "odd" is a market rate of return from investment. Since an odd system may be rather either "multiple to 1" or "multiple to multiple" than "1 to 1" in a stock market, the Kelly criterion discussed just above is invalid in general. So, we need to derive another form of the Kelly criterion which is appropriate for case which the odd system is "V to 1" for explanation in this paper. The odd system supposed here implies that when A wins a game, he will receive "V" multiple of his betting. Otherwise, he will lose an entire amount of money he bet on the game. The mathematical expressions for "$G(f)$" and "$g(f)$" are given in Equation (10) and (11), respectively (Kelly 1956, Thorp 1971, Thorp 1997).

$$G(f) = \ln(X_N / X_0) = W \ln(1+Vf) + L \ln(1-f) \tag{10}$$

$$g(f) = p \ln(1+Vf) + q \ln(1-f) \tag{11}$$

Then, we will come by the following Equation (12) for "f^*" for the odd system.

$$f^* = \frac{p(V+1)-1}{V} \tag{12}$$

The term of "f^*" in Equation (12) is value to maximize Equation (11). Substituting Equation (11) for Equation (12), then we get Equation (13).

$$g(f^*) = p \ln(V+1) + q \ln\{\frac{q+(1+V)}{V}\} \tag{13}$$

As discussed above, Equation (13) says that an investor or game player may attain a maximum geometric rate of return in investments or games or a growth rate of an investment capital. To apply the results derived above for a stock investment, we need to collect and analyze historical stock prices, calculate a probability that a stock price increased and an average winning and loss ratio, and finally determine a gain per unit bet. Taking these things into Equation (11) and (12) in terms of a stock investment, we will end up with the following Equation (14) and (15).

$$g(f) = p \ln(1+WLf) + q \ln(1-f) \tag{14}$$

$$f^* = p - \frac{(1-p)}{WL} = \frac{p(WL+1)-1}{WL} \tag{15}$$

where,

p : a probability that a stock price increase,
WL: Win/Loss ratio or a betting ratio where

$$\frac{an\quad average\quad return\quad when\quad a\quad stock\quad price\quad increase}{an\quad averge\quad return\quad when\quad a\quad stokc\quad price\quad decrease}.$$

Substituting Equation (14) with Equation (15), it becomes Equation (16).

$$g(f^*) = p\ln(WL+1) + q\ln\{\frac{q+(1+WL)}{WL}\} \tag{16}$$

When investment is made with " f^* " of the investment capital accumulated by so far, Equation (16) says that an investor will earn a maximum geometric rate of return in his/her investment or gambling games.

5. A Numerical Example with the Stock listed in KOSPI 200

As described above, the main objective of the paper is to show how to construct an optimal portfolio based on the Kelly criterion. To accomplish the objective, we collected data on the weekly stock prices of the companies listed in KOSPI 200 from Jan. of 1986 to Dec. of 2012. We also collected the KOSPI 200 weekly indices for the same period of time.

5.1 Data Collection and Calculation of Statistics

The data in <Table 1> was collected on the stock prices of the companies listed in KOSPI 200 from the date having been provided to the participants in the stock investment contest by Woori Investment Company (http://www.woriwm.com). We calculated a weekly average rate of return for each company, a probability(p) that a stock price increased, a winning and loss ratio, and a Win/Loss ratio. <Table 1> shows a partial portion of what was done above.

Table 1: The basic data on the performance of a stock

No.	Stock Code	Stock Name	p	Win Ratio	Loss Ratio	*W/L*
1	138930	BSFNG	0.4651	1.0388	0.9617	1.0124
2	001040	CJ	0.5070	1.0458	0.9562	1.0454
3	000120	CJKOR	0.4830	1.0578	0.9498	1.1511
4	097950	CJJ	0.5185	1.0417	0.9585	1.0035
5	078930	GS	0.4966	1.0469	0.9584	1.1270
6	006360	GSC	0.5110	1.0547	0.9468	1.0289
7	105560	KB	0.5163	1.0407	0.9545	0.8944
8	002380	KCC	0.4941	1.0498	0.9566	1.1474
9	025000	KPXC	0.4901	1.0476	0.9559	1.0791
10	030200	KT	0.4937	1.0355	0.9656	1.0312
11	033780	KT&G	0.5045	1.0307	0.9711	1.0629
12	003550	LG	0.5056	1.0503	0.9518	1.0437
13	034220	LGDIS	0.5218	1.0431	0.9527	0.9113
14	001120	LGT	0.5066	1.0515	0.9505	1.0401
15	068870	LGL	0.5178	1.0461	0.9540	1.0017
16	051900	LGH	0.5481	1.0436	0.9606	1.1056
17	032640	LGU	0.4904	1.0491	0.9534	1.0531
18	011070	LGEN	0.5065	1.0541	0.9503	1.0879
19	066570	LGE	0.5264	1.0425	0.9539	0.9218
20	093050	LGF	0.5160	1.0448	0.9553	1.0001
⋮						
191	11210	HDWIA	0.5778	1.0506	0.9542	1.1060
192	004020	HDS	0.5008	1.0532	0.9509	1.0839
193	009540	HDH	0.5066	1.0547	0.9476	1.0421
194	003450	HDI	0.4960	1.0619	0.9403	1.0363
195	005380	HDA	0.5051	1.0476	0.9548	1.0532
196	010520	HDHS	0.4804	1.0619	0.9439	1.1031
197	010690	WHA	0.5052	1.0582	0.9433	1.0272
198	004800	HS	0.4909	1.0553	0.9494	1.0919
199	093370	WHS	0.5097	1.0647	0.9406	1.0887
200	069260	HUC	0.5249	1.0448	0.9597	1.1126

5.2 Applying the Kelly Criterion to a Stock of each Company

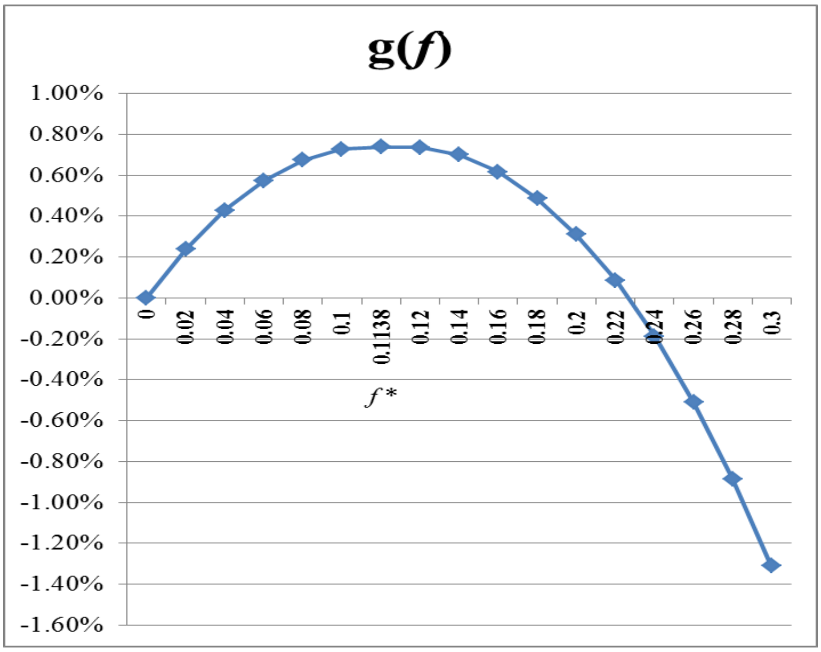
Before constructing an optimal portfolio, we applied the Kelly criterion to a stock of each company. To see the effectiveness of the Kelly criterion, we calculated the average rate of return on a stock investment of each company and compared it with the rate of return derived using the Kelly criterion. <Table 2> shows the partial portion of the results of the work. Investigating the < Table 2>, we may observe that all the rates of return derived by the Kelly criterion are greater than those derived based on the historical date. The latter case was implemented in a traditional investing strategies that most of investors used to take. Based on the observation, it can be concluded that the Kelly criterion generates a superior investment performance over the traditional investment strategies.

<Figure 1> shows the result of the sensitivity analysis of $g(f)$ as the value of f varies when an investor invests in Samsung Electronics Company listed in KOSPI 200. Carefully scrutinizing the graph, it can be conceived that the investor gets completely ruined if he/she puts approximately 0.23 of the f value and he/she maximizes his/her investment capital at f^* = 0.1138.

Table 2: The comparison of the rates of return derived based on the historical data and the applying the Kelly criterion

No.	Stock Code	Stock Name	f^*	$g(f^*)$,%	Mean,%	$g(f^*)-$ Mean,%
1	138930	BSFNG	-0.0334	0.0601	-0.2318	0.2919
2	001040	CJ	0.0768	0.3373	0.1561	0.1812
3	000120	CJKOR	0.1164	0.9444	0.1875	0.7569
4	097950	CJJ	0.0788	0.3398	0.1589	0.1809
5	078930	GS	0.0813	0.3993	0.2304	0.1688
6	006360	GSC	0.0814	0.3765	0.1897	0.1868
7	105560	KB	0.0170	0.0139	-0.0978	0.1117
8	002380	KCC	0.0911	0.5172	0.2539	0.2633
9	025000	KPXC	0.0586	0.2026	0.0811	0.1215
10	030200	KT	0.0359	0.0713	0.0098	0.0615
11	033780	KT&G	0.0611	0.2086	0.1140	0.0946
12	003550	LG	0.0742	0.3144	0.1529	0.1615
13	034220	LGDIS	0.0368	0.0668	-0.0121	0.0789
14	001120	LGT	0.0753	0.3234	0.1603	0.1632
15	068870	LGL	0.0796	0.3477	0.1655	0.1822
16	051900	LGH	0.1677	1.6626	0.5911	1.0715
17	032640	LGU	0.0586	0.2018	0.0309	0.1709
18	011070	LGEN	0.0929	0.5128	0.2861	0.2267
19	066570	LGE	0.0505	0.1270	0.0530	0.0740
20	093050	LGF	0.0676	0.2460	0.1413	0.1047
⋮						
191	11210	HDWIA	0.2310	3.2284	0.9307	2.2977
192	004020	HDS	0.0838	0.4188	0.2075	0.2113
193	009540	HDH	0.0758	0.3281	0.1780	0.1501
194	003450	HDI	0.0633	0.2328	0.0581	0.1747
195	005380	HDA	0.0737	0.3105	0.1641	0.1464
196	010520	HDHS	0.0779	0.3893	0.0562	0.3331
197	010690	WHA	0.0752	0.3246	0.1329	0.1917
198	004800	HS	0.0687	0.2838	0.1327	0.1511
199	093370	WHS	0.1160	0.8329	0.3790	0.4540
200	069260	HUC	0.1313	1.0343	0.4295	0.6048

Figure 1: The sensitivity analysis of $g(f)$ as f varies



5.3 The Construction of the Optimal Portfolio

Since the main objective of the Kelly criterion is to maximize the geometric expectation of the investment capital by determining an optimal allocation of the investment capital, we can put a maximization of a portfolio value as an objective function of a nonlinear programming subject to three constraints as followings:

$$Maximize \quad \sum_{i=1}^{N} f_i R_i + r(1 - \sum_{i=1}^{N} f_i)$$

Subject to

$$\sum_{i=1}^{N} f_i \leq 1$$
$$f_i \leq f_i^*$$
$$R_i = p_i \ln(1 + WL_i f_i) + (1 - p_i)\ln(1 - f_i)$$
$$0 \leq f_i \leq 1, \quad for \quad i = 1, 2, \ldots N$$

where,

N : a number of the stocks involved in the portfolio,
f_i : a fraction of an investment capital for stock i,
R_i : an average rate of return for stock i,
r : a risk-free rate of return on a one-year government bond,
f_i^* : an optimal fraction of an investment capital for stock i,
p_i : a probability that the price of stock i increases,
WL_i : Win/Loss ratio for stock i.

The objective function is concerned with maximizing the value of the portfolio which consists of a risk and risk-free investment. It means that when an investor allocates only a partial portion of his/her whole investment capital, there remains some portion not to be invested in a risky investment. In this case, the general assumption which may be practical is that the rest of the investment capital is put in risk-free investment alternatives such government bonds. The first constraint implies that a sum of all allocation ratios is greater than or equal to 1 with the assumption that a borrowing and short-selling activities are not allowed. The second constraint shows the value of an optimal fractional allocation of the investment capital which is determined inside the programming. As seen in <Figure 1>, the value of $g(f)$ increases until the value of f approaches f^*, and it becomes maximum when the value of f reaches at f^*. Thereafter, it begins to go down penetrating 0 and to in infinity. Therefore, it is necessary to limit the value of f within f^* which is expressed in the third constraint. The fourth constraint simply says that all fractional value must be greater than or equal to 0.

5.4 A Numerical Example with a Real Data

In this section, we implemented the nonlinear programming suggested in section 4.3 to form an optimal portfolio of the stocks which were listed in KOSPI 200. We took a two-steps approach to demonstrate an efficacy of the Kelly criterion and to see what would happen to a performance of each stock. To carry out the work, we first blindly chose top 5 stocks out of KOSPI 200 and made up of the portfolio. After carrying out the work, it was recommended to check out if there was a remnant of the investment capital which was usually invested in risk-free investments. If there was, it was required to form another portfolio which supposed to fully absorb the investment capital. So, we expanded a portfolio consisting of top 10 stocks out of KOSPI 200. The results are shown in <Table 3> and <Table 4>. Looking at <Table 3>, we can recognize that the performance of the top 5 stocks generated far greater rate of return than a historical rate of return by 0.93% weekly. Also, there is no stock whose rate of return is less than the historical rate of return. When we performed a portfolio analysis with the top 10 stocks in KOSPI 200, it produced greater rate of return than that with the top 5 stock in <Table 3> by 0.096% weekly and much greater rate of return than the historical rate of return by 1.026% weekly. Based on the analysis of the performance of the portfolio with the Kelly criterion, it can be said with caution and limitation at this time that when an investor follows the investment strategy with the Kelly criterion, he/she may achieves greater rate of return than with an ordinary or historical strategy.

Table 3: The result of the portfolio with top 5 stocks in KOSPI 200

Rank	Stock Name	f	r
1	Hyudai_Wia	19.601%	2.125%
2	Hansae	14.861%	1.344%
3	SK C&C	14.001%	1.168%
4	LG_household & healthcare	13.930%	1.071%
5	LG_Chem	10.858%	0.615%
6	Govern_Bond	26.749%	0.059%
Weekly expected rate of return with the Kelly criterion		1.011%	
Weekly expected rate of return with KOSPI 200		0.081%	

Table 4: The result of the portfolio with top 10 stocks in KOSPI 200

Rank	Stock Name	f	r
1	Hyudai_Wia	19.601%	2.125%
2	Hansae	14.861%	1.344%
3	SK C&C	14.001%	1.168%
4	LG_household & healthcare	13.930%	1.071%
5	LG_Chem	10.858%	0.615%
6	Kepco-E&C	0.000%	0.000%
7	Poongsan	8.211%	0.333%
8	Orion	0.420%	0.032%
9	Huchems	9.789%	0.532%
10	Samsung-fire	8.328%	0.379%
11	Govern_Bond	0.000%	0.000%
Weekly expected rate of return with the Kelly criterion		1.107%	
Weekly expected rate of return with KOSPI 200		0.081%	

6. Concluding Remarks

If there exists 100% of probability that an expected rate of return is obtained and greater than or equal to 0, the best investment strategy is to put all the investment capital into the investment asset(s) to maximize his/her wealth. However, since all investment environment is uncertain such investment strategy is not valid and efficient. Then, even when a rate of return on risky investment asset is to be judged tremendously high, what is practical in reality is to divide the investment capital into two parts: some of which is invested in risky assets and the rest of which in risk-free assets. An investment strategy to demonstrate such an investment strategy as we have seen is called the Kelly criterion. A group of the investors who had accomplished a high rate of return using the investment strategy of the Kelly criterion includes Shannon, Thorp, Ziemba, and Buffet. We demonstrated a way to construct an optimal portfolio consisting of the stocks listed in KOSPI 200 based on the Kelly criterion. And it was concluded that investing in the stocks using the Kelly criterion generated much higher rate of return on the portfolio than a historical and traditional investing strategies. In this paper, we show the nonlinear programming for the portfolio using Kelly criterion. In case of Markowitz theory, there are so many studies with linear programming to develop the critical line algorithm, thus as a further research we will develop more practical, sophisticated, and linearized version of an original nonlinear programming shown in this paper utilizing the these studies.

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