World Journal of Social Sciences
Vol. 1. No. 3. July 2011. Pp. 100-115

Moral Hazard Resolved in Communication Network

Takashi Matsuhisa* and Dianyu Jiang**

This paper investigates the role of communication among the principal and agents under uncertainty. We treat the problem: How epistemic conditions of communication will be able to settle a moral hazard in team in the principal-agent model under uncertainty. We shall propose a communication process to resolve the moral hazard in the principal-agent model by communication. We assume that the agents have the knowledge structure induced from a binary relation associated with the multi-modal logic ***S5****n. We show that the moral hazard can be resolved if the principal and each agent communicate their expected marginal costs according to the communication graph.*

Keywords: Agreeing to disagree, Communication, Information, Knowledge revision, Message, Moral Hazard, Principal-agent model under Uncertainty

JEL Codes: C02, L20 and M51

1. Introduction

This article re-examines a principal-agent model with moral hazard from the epistemic point of view. It highlights communication process between the principal and the agents, and the purpose is to give a possible resolution of the moral hazard. Let us consider that there are the principal and agents more than two in a firm. The principal makes a profit by selling the productions made by the agents with paying their costs. Assume that both the principal and agents aim to maximise each gross return independently. The moral hazard arises that there is not the sharing rule so that the principal makes a contract with every agent such that the total amount of all profits is refunded to each agent in proportion to the agent's contribution to the firm, i.e.; the agents' expected costs are not equal for the principal and the agents.

To resolve this phenomenon we shall extend the model to the principal agents model with incomplete information, where the principal and the agents have the knowledge structure induced from a binary relation associated with the multi-modal logic **S5**n.

We focus on the situation that the principal and the agents communicate their expected costs as messages according through the communication graph.

---

* Takashi Matsuhisa, Department of Natural Sciences, Ibaraki National College of Technology, Nakane 866, Hitachinaka-shi, Ibaraki 312-8508, Japan. Email : mathisa@ge.ibaraki-ct.ac.jp
**Dianyu Jiang, Institute of Games Theory with Applications, Huaihai Institute of Technology, Lianyungang, 222005, China. E-mail: jiangdianyu425@126.com

Where each agent and the principal communicate privately each other about their expected marginal cost as messages: The principal sends his expected marginal cost as messages. The agent as the recipient of the message revises his/her information structure and recalculates the expected marginal cost under the revised information structure, and he/she sends the revised expected marginal cost to the principal. The principal as the recipient of the message revises his/her information structure and recalculates the expected marginal cost under the revised information structure, and he/she sends the revised expected marginal cost to the agent, and so on.

Our result is

Main theorem. *In the circumstance the limiting expected marginal costs actually coincide.*

In words, the principal and the agents can reach consensus on their expected marginal costs for their jobs if the principal and each agent communicate their expected marginal costs according to the protocol.

2. Literature Review

The first formal analysis of the principal-agent relationship and the phenomena of moral hazard was made by Arrow (1963), and especially, the treatment of the principal-agent model with many sided moral hazard was given by Holmstrom (1982). The many sided moral hazard can arise when there are many agents that affect gross returns and their individual actions are not observed by each other. This is formalised as the issue in a partnership model whether there exist any sharing rules that both balances the budget and under which an efficient action is a Nash equilibrium.

Holmstrom (1982) and Williams and Radner (1989) respectively analysed the conditions for existing the sharing rule such that some actions profile satisfies the first-order conditions for an equilibrium.

This article focuses on knowledge on the agents' costs but not their actions, and so we treat the special type of principal-agent model with the many sided moral hazard.

Aumann (1976) introduced the formal notion of common-knowledge in the partition information structure in the game theoretical framework, and showed his famous “Agreeing to disagree” theorem: if all agents commonly known their posterior, then all the posteriors must be equal each other. Bacharach (1985) extended the Aumann’s theorem into the **S5**n-knowledge model.

Krasucki (1996) extended a version of “Agreeing to disagree” theorem in the communication model introduce by Parikh and Krasucki (1990): He showed that after long run communication among agents, all the limiting values of sequence of reivions of posteriors must be equal.

This article highlights the epistemic aspect of the principal and agents to analyse the moral hazard problem. It is new to treat the problem, and we propose the principal-agents model with knowledge structure following Parikh and Krasucki (1990), and we give a possible resolution of moral hazard by communication in this framework.

3. Moral Hazard

Let us present the principal-agent model formally: There are the principal P and *n* agents $N=\{1,2,\cdots,n\}$ ($n\geq 2$) in a firm. Denote $\overline{N}=\{P\}\cup N$, and $k \in \overline{N}$.

The principal makes a profit by selling the productions made by the agents. He/she makes a contact with each agent k that the total amount of all profits is refunded to each agent k in proportion r_k to the agent's contribution to the firm. Let e_k denote the measuring managerial effort for k's productive activities. The set of possible efforts for k is denoted by E_k with $E_k \subseteq R$.

Let $I_P(\cdot)$ be a real valued continuously differentiable function on E_k. It is interpreted as the profit by selling the productions made by the agent *k* with the cost $c(e_k)$. Here we assume $I_k'(e_k)>0$ and the cost function *c*(・) is a real valued continuously differentiable function on $E=\bigcup_{k\in\overline{N}} E_k$. Let I_P be the total amount of all the profits that is defined by

$$I_P(e)=I_P(e_1,e_2,\cdots;e_n)=\sum_{k\in\overline{N}} I_k(e_k)$$

Each agent can observe his/her own effort e_k, but the principal P cannot observe it, and thus P does not obtain the total amount $I_P(e)$ exactly. The principal will view each e_k as a random variable $e_k:\Omega\rightarrow\mathbb{R}$ on a probability space (Ω,μ), and he/she can obtain the expectation of $I_P(e)$:

$$Exp[I_P(e)]=\sum_{k=1}^{n} Exp[I_k(e_k)]=\sum_{\omega\in\Omega} I_k(e_k(\omega))\mu(\omega)$$

The principal has his/her expectation of the pure profit given by

$$Exp\,[I_P(e)]-\sum_{k=1}^{n} c(e_k)$$

and so the optimal plan for the principal then solves the following problem:

$$Max_{e=(e_1,e_2,\cdots,e_n)}\{Exp[I_P(e)]-\sum_{k=1}^{n}c(e_k)\}$$

The necessity conditions for critical points is that the partial derivatives of the above with respect to e_k must be *0* for each every $k=1,2,\cdots,n$, and thus we obtain

$$\frac{\partial}{\partial e_k}Exp[I_k(e_k)]-c'(e_k)=0 \qquad (1)$$

Let $W_k(e_k)$ be the total amount of the refund to agent k:

$$W_k(e_k)=r_k I_k(e_k) \text{ with } \sum_{k=1}^{n}r_k=1 \text{ and } 0<r_k<1,$$

where r_k denotes the proportional rate representing k's contribution to the firm.

Each agent k cannot observe the others' efforts e_j with $j\neq k$, and he/she can have the expectation of $W_k(e_k)=r_k I_k(e_k)$ which is given by

$$Exp[W_k(e_k)]=r_k Exp[I_k(e_k)],$$

so the optimal plan for agent k solves the problem: For every $k=1,2,\cdots,n$,

$$Max_{e_k}\{Exp[W_k(e_k)]-c_k(e_k)\} \text{ subject to } \sum_{k=1}^{n}r_k=1 \text{ with } 0<r_k<1.$$

We assume that r_k is independent of e_k, and the necessity condition for critical points is also that

$$\frac{\partial}{\partial e_k}Exp[W_k(e_k)]-c'(e_k)=0$$

and thus

$$r_k\frac{\partial}{\partial e_k}Exp[I_k(e_k)]-c'(e_k)=0. \qquad (2)$$

In viewing $0<r_k<1$, it follows that Eq. (2) is in a contradiction to Eq. (1). This contradictory situation is called the **moral hazard** in the principal-agent model.

4. The Model

4.1 Information and Knowledge

By *partition information structure* we mean $\langle \Omega, (\Pi_i)_{i \in \bar{N}} \rangle$ in which Ω is a non-empty finite state-space and Π_i is i's information function

$$\Pi_i : \Omega \to 2^{\Omega}, \omega \mapsto \Pi_i(\omega)$$

satisfying the three postulates: For each $i \in \bar{N}$ and for any $\omega \in \Omega$,

(Ref) $\omega \in \Pi_i(\omega)$
(Trn) $\xi \in \Pi_i(\omega)$ implies $\Pi_i(\xi) \subseteq \Pi_i(\omega)$
(Sym) If $\xi \in \Pi_i(\omega)$ then $\omega \in \Pi_i(\xi)$.

This structure is equivalent to the Kripke semantics for the multi-modal logic **S5**n. The set $\Pi_i(\omega)$ will be interpreted as the set of all the states of nature that i knows to be possible at ω, or as the set of the states that i cannot distinguish from ω. We call $\Pi_i(\omega)$ i's *information set* at ω.

We will give the formal model of *knowledge* as follows (C.f.; Fagin et al~1995.)

Definition. The **S5**n-*knowledge structure* is a tuple $\langle \Omega, (\Pi_i)_{i \in \bar{N}}, (K_i)_{i \in \bar{N}} \rangle$ consisting of a partition information structure $\langle \Omega, (\Pi_i)_{i \in \bar{N}} \rangle$ and a class of i's *knowledge operator* $K_i : 2^{\Omega} \to 2^{\Omega}, E \mapsto K_i(E)$ defined by

$$K_i(E) = \{\omega \in \Omega \mid \Pi_i(\omega) \subseteq E\}.$$

The event $K_i(E)$ will be interpreted as the set of states of nature for which i knows E to be possible.

We record the properties of i's knowledge operator: For every $E, F \in 2^{\Omega}$,

N $K_i(\Omega) = \Omega$;
K $K_i(E \cap F) = K_i(E) \cap K_i(F)$;
T $K_i(E) \subseteq E$;
4 $K_i(E) \subseteq K_i(K_i(E))$;
5 $\Omega \setminus K_i(E) \subseteq K_i(\Omega \setminus K_i(E))$

According to these properties we can say the structure $\langle \Omega, (\Pi_i)_{i \in \bar{N}}, (K_i)_{i \in \bar{N}} \rangle$ is a model for the multi-modal logic **S5**n.

4.2 System Design Function

Let Z be a set of decisions, which set is common for all agents. By a *system design function* (or simply, a *decision function*) we mean a mapping $f : 2^{\Omega} \times 2^{\Omega} \to Z$.

We refer the following property of f: Let X be an event.

DUC (Disjoint Union Consistency): *For every pair of disjoint events* S *and* T*, if* $f(X;S) = f(X;T) = d$ *then* $f(X;S \cup T) = d$;

PUD (Preserving Under Difference): *For all events* S *and* T *such* $S \subseteq T$*, if* $f(X;S) = f(X;T) = d$ *then* $f(X;T - S) = d$.

CNV (Convexity): *For every pair of disjoint events* E *and* F*, there are positive numbers* $\lambda, \delta \in [0,1]$ *such that* $f(X; E \cup F) = \lambda f(X;E) + \delta f(X;F)$ *with* $\lambda + \delta = 1$.

By i's *decision function* associated with a system design function f under agent i's private information we mean the function $d_i : 2^{\Omega} \times \Omega \to Z$ defined by $d_i(X;\omega) = f(X; \Pi_i(\omega))$, it is called i's decision value of X associated with f under agent i's private information at ω.

If f is intended to be a posterior probability, we assume given a probability measure μ on Ω which is common for all agents. Precisely, for some event X, Y, $f(X;Y)$ is given by the conditional probability under private information: $f(X;Y) = \mu(X \mid Y)$. Then the i's decision value of X is the conditional probability value $d_i(X;\omega) = \mu(X; \Pi_i(\omega))$.

4.3 Principal-Agent Model under Uncertainty

Let us reconsider the principal-agent model and let notations and assumptions be the same in Section 2.

We require additional assumptions **A1-2** below. These play essential role to resolve the moral hazard problem.

Let us denote by $[e_k(\omega)]$ the event: $[e_k(\omega)] = \{\xi \in \Omega \mid e_k(\xi) = e_k(\omega)\}$ for $k \in N$, and by $[e(\omega)]$ the event: $[e(\omega)] = \bigcap_{k \in N} [e_k(\omega)]$.

A1 The principal P has a partition information function $\Pi_P : \Omega \to 2^{\Omega}$ and each agent k has also his/her partition information function $\Pi_k : \Omega \to 2^{\Omega}$.

A2 There exists the decision function $f : 2^{\Omega} \times 2^{\Omega} \to \mathbb{R}$ satisfying the conditions: For each $\omega, \xi \in \Omega$,

(a) $f(\,[e(\xi)\,];\Pi_P(\omega)\,) = \dfrac{\partial}{\partial e_k(\xi)} Exp[I_k(e_k) \mid \Pi_P(\omega)\,].$

(b) $f(\,[e(\xi)\,];\Pi_k(\omega)\,) = \dfrac{\partial}{\partial e_k(\xi)} Exp[W_k(e_k) \mid \Pi_k(\omega)\,]$

Definition: The structure

$$M = \left\langle \overline{N}, (I_k)_{k \in \overline{N}}, (e_k)_{k \in N}, c, (r_k)_{k \in N}, \Omega, \mu, (\Pi_i)_{k \in \overline{N}}, f \right\rangle$$

is called a principal-agent model under uncertainty where μ is a probability measure on Ω.

The optimal plans for principal P and agent k are then to solve the following maximization problems:

[PE] $\quad Max_{e=(e_1, e_2, \cdots, e_n)} \{Exp[I_P(e) \mid \Pi_P(\omega)\,] - \sum_{k=1}^{n} c(e_k)\}$

[AE] For $k \in N$,

$Max_{e=(e_1, e_2, \cdots, e_n)} \{Exp[W_k(e) \mid \Pi_k(\omega)\,] - c(e_k)\,\}$ subject to $\sum_{k=1}^{n} r_k = 1$ with $0 < r_k < 1.$

From the necessity condition for critical points together with **A2** it can be seen by [PE] that the principal's marginal expected costs for agent k will be given by

$$c'_P(e_k(\xi);\omega) = f(\,[e_k(\xi)\,];\Pi_P(\omega)\,),$$

and it also follows from **AE** that agent k's expected marginal costs is given by

$$c'_k(e_k(\xi);\omega) = f(\,[e_k(\xi)\,];\Pi_k(\omega)\,).$$

To establish our solution program we have to solve the problem: Construct the information structure together with decision function such that the above conditions **A1** and **A2** are true.

Under these circumstances, a resolution of the moral hazard for **S5**n-knowledge mode will be restate as follows: There exists a sequence of states $\{\omega_0, \omega_1, \omega_2, \cdots, \omega_m\} \subset \Omega$ such that $c'_P(\cdot, e(\omega_0)) \neq c'_1(\cdot, e(\omega_1)) \neq c'_2(\cdot, e(\omega_2)) \neq \cdots \neq c'_n(\cdot, e(\omega_m)).$

5. Communication

In this section we present the communication process in the line of Parikh and Krasucki (1990).

We assume that the principal and the agents communicate by sending *messages*. Let T be the time horizontal line $\{0,1,2,\cdots,t,\cdots\}$

A *protocol* is a mapping $\Pr: T \to \overline{N}\times\overline{N}, t \mapsto (s(t), r(t))$ such that $s(t) \neq r(t)$. Here t stands for *time* and $s(t)$ and $r(t)$ are, respectively, the *sender* and the *recipient* of the communication which takes place at time t.

We consider the protocol as the directed graph whose vertices are the set of all agents and principal, $\overline{N}$ and such that there is an edge (or an arc) from $i \in \overline{N}$ to $j \in \overline{N}$ if and only if there are infinitely many $t \in T$ such that $i = s(t)$ and $j = r(t)$

A protocol is said to be *fair* if the graph is strongly connected; in words, every player in this protocol communicates directly or indirectly with every other player infinitely many often. It is said to contain a *cycle* if there are $i_1, i_2, i_3, \cdots, i_l, \cdots, i_k$ with $k \geq 3$ such that for all $l \leq k$, i_l communicates directly with i_{m+1}, and such that i_k communicates directly with i_1. The communication is assumed to proceed in *rounds*; that is, there exists a time $m \in T$ such that for all $t \in T$, $\Pr(t+m) = \Pr(t)$. The *period* of the protocol is the minimal number of all $m \in T$ such that for every $t \in T$, $\Pr(t+m) = \Pr(t)$.

The protocol is said to be *acyclic* if the graph has no cycle. A protocol is said to be *fair* if the graph is strongly-connected; in words, every $i \in \overline{N}$ in this protocol communicates directly or indirectly with every other $j \in \overline{N}$ infinitely often. It is said to be *acyclic* if the graph contains no cyclic path.

Definition: A *communication process* π of revisions of the values of decision function is a triple $\pi = \langle \Pr, (Q_i^t)_{(i,t)\in\overline{N}\times T}, f \rangle$ consisting of

1. $\Pr(t) = (s(t), r(t))$ is a fair protocol such that for every t, $r(t) = s(t+1)$, and it communications proceed in *rounds*; i.e., there exists a natural number m such that for all t, $s(t+m) = s(t)$ and
2. $Q_i^t : 2^\Omega \times \Omega \to 2^\Omega$ is defined inductively in the following way:
   (i) We assume given a mapping $Q_i^0(\cdot;\omega) = \Pi_i(\omega)$.
   (ii) Suppose $Q_i^t(\cdot;\omega)$ is defined. Let $d_i^t(X;\omega)$ denote the decision value

$$d_i^t(X;\omega) = f(X; Q_i^t(X;\omega)),$$

and let $m_i^t(X;\omega)$ be the message sent by $s(t)$ to $r(t)$ defined by

$$m_i^t(X;\omega) = \{\xi \in \Omega \mid d_i^t(X;\xi) = d_i^t(X;\omega)\}$$

(iii) Then $Q_i(X;\omega)$ is defined as follows:

- $Q_i^{t+1}(X;\omega) = Q_i^t(X;\omega)$ if $i \neq r(t)$
- $Q_i^{t+1}(X;\omega) = Q_i^t(X;\omega) \cap m_j^t(X;\omega)$ if $(j,i) = (s(t), r(t))$

Specifically the sender j sends to j the message that his decision is $d_j^t(X;\omega)$.

Because Ω is finite, the descending chain $\{Q_i^t(X;\cdot) \mid t = 0,1,2,\cdots\}$ is finite, so it must be stationary and the limit $Q_i^\infty(X;\cdot)$ exists in each state.

We denote $d_i^\infty(X;\omega) = f(X;Q_i^\infty(X;\omega))$ called the *limiting decision* of f about X at ω for i.

We say that *consensus* on the limiting decisions of a system design function f can be guaranteed if there exists a sequence of states $\{\omega_0, \omega_1, \omega_2, \cdots, \omega_m\} \subset \Omega$ such that $d_P^\infty(X;\omega_0) = d_1^\infty(X;\omega_1) = d_2^\infty(X;\omega_2) = \cdots = d_n^\infty(X;\omega_n)$.

6. Moral Hazard Resolved by Communication

This section investigates the moral hazard problem from the communication model.

Let us reconsider the principal-agents model and let notations and assumptions be the same in Section 2.

6.1 Principal-Agents Model with Communication

We have already set up the principal-agents model under uncertainty:

$$M = \langle \overline{N}, (I_k)_{k \in \overline{N}}, (e_k)_{k \in N}, c, (r_k)_{k \in N}, \Omega, \mu, (\Pi_i)_{k \in \overline{N}}, f \rangle$$

Definition. By the *principal-agents model with communication* we mean the structure

$$M_C = \langle \overline{N}, (I_k)_{k \in \overline{N}}, (e_k)_{k \in N}, c, (r_k)_{k \in N}, \Omega, \mu, (Q_i^t)_{(i,t) \in \overline{N} \times T}, f, \Pr \rangle$$

in which the principal and each agent communicates their expected marginal costs through messages according to the protocol $\Pr$ and $\pi = \langle \Pr, (Q_i^t)_{(i,t) \in \overline{N} \times T}, f \rangle$ is a communication process satisfying the additional conditions as below: We assume given a mapping $Q_i^0(\cdot;\omega) = \Pi_i(\omega)$. Suppose $Q_i^t(\cdot;\omega)$ is defined. The message $m_i^t(\cdot;\omega)$ given by

- $m_P^t([e(\xi)];\omega) = \{\xi \in \Omega \mid d_P^t([e(\xi)];\xi) = d_P^t([e(\xi)];\omega)\}$ if $\Pr(t) = (P,k)$
- $m_k^t([e(\xi)];\omega) = \{\xi \in \Omega \mid d_k^t([e(\xi)];\xi) = d_k^t([e(\xi)];\omega)\}$ if $\Pr(t) = (k,l)$ for $k \in N, l \in \overline{N}$

Then $Q_i^t(X;\omega)$ is defined as follows:

i. $Q_i^{t+1}([e_k(\xi)];\omega) = Q_i^t([e_k(\xi)];\omega)$ if $i \neq r(t)$

ii. $Q_i^{t+1}([e_k(\xi)];\omega) = Q_i^t([e_k(\xi)];\omega) \cap m_j^t([e_k(\xi)];\omega)$ if $(j,i) = (s(t), r(t))$

and

A2^t There exists the decision function $f : 2^\Omega \times 2^\Omega \to \mathbb{R}$ satisfying the conditions: For each $\omega, \xi \in \Omega, t \in T$,

(a) $f([e(\xi)]; Q_P^t(\{\xi\};\omega)) = \dfrac{\partial}{\partial e_k(\xi)} Exp[I_P(e) \mid Q_P^t(\{\xi\};\omega)]$.

(b) For $k \in N$,

$$f([e(\xi)]; Q_k^t(\{\xi\};\omega)) = \frac{\partial}{\partial e_k(\xi)} Exp[W_k(e_k) \mid Q_k^t(\{\xi\};\omega)]$$

At each t the optimal plans for principal P and agent $k \in N$ are then to solve the maximization problem:

[PE^t] $\quad Max_{e=(e_1,e_2,\cdots,e_n)} \{Exp[I_P(e) \mid Q_P^t(\{\xi\};\omega)] - \sum_{k=1}^{n} c(e_k)\}$

[AE^t] For $k \in N$,

$Max_{e=(e_1,e_2,\cdots,e_n)} \{Exp[W_k(e_k) \mid Q_k^t(\{\xi\};\omega)] - c(e_k)\}$ subject to $\sum_{k=1}^{n} r_k = 1$ with $0 < r_k < 1$.

From the necessity condition for critical points together with **A2^t** it can be seen by [PE^t] that the principal's marginal expected costs for agent k will be given by

$$c_P'^t(e_k(\xi);\omega) = f([e(\xi)]; Q_k^t(\{\xi\};\omega)),$$

and it also follows from [AE^t] that agent k's expected marginal costs is given by

$$c'^{t}_{k}(e_k(\xi);\omega) = f(\{e_k(\xi)\}, Q^t_k(\{\xi\};\omega)).$$

Because Ω is finite, the descending chain $\{Q^t_i(\cdot;\omega) \mid t = 0,1,2,\cdots\}$ is finite, it must be stationary, and so the limit $Q^\infty_i(\cdot;\omega)$ exists at each state. Therefore we denote the *limiting marginal expected cost* $c'^{\infty}_{k}(e_k(\xi);\omega) = \lim_{t\to\infty} c'^{t}_{k}(e_k(\xi);\omega)$ and $c'^{\infty}_{P}(e_k(\xi);\omega) = \lim_{t\to\infty} c'^{t}_{P}(e_k(\xi);\omega)$.

Under these circumstances, a resolution of the moral hazard (Maun theorem) will be restated as follows:

Theorem 1. *For the principal-agents model with communication the principal and all agents can reach consensus on their limiting marginal expected costs: There exists a sequence of states* $\{\omega_0, \omega_1, \omega_2, \cdots, \omega_n\} \subset \Omega$ *such that*

$$c'^{\infty}_{P}(e_k(\xi);\omega_0) = c'^{\infty}_{1}(e_k(\xi);\omega_1) = c'^{\infty}_{2}(e_k(\xi);\omega_2) = \cdots = c'^{\infty}_{n}(e_k(\xi);\omega_n) \text{ for } k \in N.$$

We shall prove the theorem by dividing the two cases: $\Pr$ is acyclic and cyclic in the following subsections.

6.2 Acyclic Communication

We consider the following scenario: Let us start the principal P and each agent k has a partition information structure $\langle \Omega, (\Pi_i)_{i\in \bar{N}} \rangle$, and also they have a common decision function $f : 2^\Omega \times 2^\Omega \to \mathbb{R}$.

The principal P sends his/her expected marginal cost $c'^{0}_{P}(\cdot, e_1(\omega_1))$ for agent 1 under his/her initial information partition $Q^0_P = \Pi_P$ to the agent 1 as message $[c'^{0}_{P}(\cdot, e_1(\omega_1))]$.

The recipient 1 refines her/his information partition Q^1_1 according to the message such as $Q^1_1(\{e_1(\xi)\};\omega) = Q^0_1(\{e_k(\xi)\};\omega) \cap m^1_P(\{e_1(\xi)\};\omega)$. She/he gets her/his revised expected marginal cost $c'^{1}_{1}(e_1(\xi);\omega) = f(\{e_1(\xi)\}, Q^1_1(\{\xi\};\omega))$, and sends it to the principal.

The principal P refines his/her information partition Q^1_P according to the message $[c'^{1}_{1}(e_1(\xi);\omega)]$; i.e., $Q^2_P(\{e_1(\xi)\};\omega) = Q^1_P(\{e_1(\xi)\};\omega) \cap m^1_P(\{e_1(\xi)\};\omega)$. He/she revises his/her revised expected marginal cost $c'^{2}_{P}(e_2(\xi);\omega) = f(\{e_2(\xi)\}, Q^2_P(\{\xi\};\omega))$, and sends $[c'^{2}_{P}(e_2(\xi);\omega)]$ as message to the agent 2.

The recipient 2 refines her/his information partition Q_2^2 according to the message $[c'^2_P(e_2(\xi);\omega)]$; i.e., $Q_2^3([e_2(\xi)];\omega)=Q_2^2([e_2(\xi)];\omega)\cap m_P^2([e_1(\xi)];\omega)$. She/he revises his/her revised expected marginal cost $c'^3_2(e_2(\xi);\omega)=f([e_2(\xi)]|Q_2^3(\xi);\omega)$, and sends $[c'^3_2(e_2(\xi);\omega)]$ to the principal.

The principal P refines his/her information partition Q_P^4 according to the message $[c'^3_2(e_2(\xi);\omega)]$; i.e., $Q_P^4([e_2(\xi)];\omega)=Q_P^3([e_2(\xi)];\omega)\cap m_2^3([e_2(\xi)];\omega)$. He/she revises his/her revised expected marginal cost $c'^4_P(e_3(\xi);\omega)=f([e_3(\xi)]|Q_P^3(\xi);\omega)$, and sends $[c'^4_P(e_2(\xi);\omega)]$ as message to the agent 3 and so on.

This protocol is given as

$$\mathrm{Pr}: P\to 1\to P\to 2\to P\to 3\to\cdots\to k\to P\to k+1\to\cdots\to P\to n\to P\to 1\to P\to\cdots$$

and it is a typical example of acyclic communications. In general,

Definition. The principal-agents model with communication is called with *acyclic* communication if Pr is given by

We can show that

Theorem 2. *If the system design function f satisfies both* **DUC** *in the principal-agent model with acyclic communication then for any $\omega,\xi\in\Omega$,*

$$c'^{\infty}_P(e_k(\xi);\omega)=c'^{\infty}_1(e_k(\xi);\omega)=c'^{\infty}_2(e_k(\xi);\omega)=\cdots=c'^{\infty}_n(e_k(\xi);\omega)\ \text{for } k\in N.$$

Proof of Theorem 2: The theorem follows immediately from the below proposition of Krasucki (1996) by setting $X=[e(\xi)]$

Proposition 1. *Let $\pi=\langle \mathrm{Pr},(Q_i^t)_{(i,t)\in\bar{N}\times T},f\rangle$ be a communication process with which the common decision function f satisfies* **DUC**. *Suppose the protocol Pr is acyclic. Then consensus on the limiting values of the decision function about an event X can be guaranteed; i.e.,*

$$d_P^{\infty}(X;\omega)=d_1^{\infty}(X;\omega)=d_2^{\infty}(X;\omega)=\cdots=d_n^{\infty}(X;\omega)$$

Proof: Let us consider the any agents $i,j\in\bar{N}$ such that if $\mathrm{Pr}(t)=(i,j)$ then $\mathrm{Pr}(t+1)=(j,i)$ for almost all $t\in T$. Let us denote $m_i(X;\omega)=m_i^{\infty}(X;\omega)$.

We can observe that $m_i(X;\omega)$ can be decomposed into the disjoint union of $Q_i^{\infty}(X;\xi)$ for $\xi\in m_i(X;\omega)\cap m_j(X;\omega)$, and that it can be also done into the disjoint union of $Q_j^{\infty}(X;\xi)$ for $\xi\in m_i(X;\omega)\cap m_j(X;\omega)$.

Therefore it follows from **DUC** that

$$f(X;m_i \cap m_j) = f(X;Q_i^\infty(X;\omega)) = f(X;Q_j^\infty(X;\omega))$$

and thus $d_i^\infty(X;\omega) = d_j^\infty(X;\omega)$.

Next we shall proceed in the general case. Because the protocol is also acyclic, we can choose the agents $k,l \in \overline{N}$ such that $\Pr(t) = (k,l), \Pr(t+1) = (l,k)$. By the same argument as above, we obtain that $d_k^\infty(X;\omega) = d_l^\infty(X;\omega)$.

Hence, by the induction argument on the period of the protocol it can be shown that $d_i^\infty(X;\omega) = d_j^\infty(X;\omega)$ for each $\omega \in \Omega$ and for all $i,j \in \overline{N}$, in completing the proof.

6.3 Cyclic Communication

Let us consider the following scenario: Let us start the principal P and each agent k has a partition information structure $\langle \Omega, (\Pi_i)_{i \in \overline{N}} \rangle$, and also they have a common decision function $f : 2^\Omega \times 2^\Omega \to \mathbb{R}$.

The principal P sends his/her expected marginal cost $c_P'^0(\cdot, e_1(\omega_1))$ for agent 1 under his/her initial information partition $Q_P^0 = \Pi_P$ to the agent 1 as message $[c_P'^0(\cdot, e_1(\omega_1))]$.

The recipient 1 refines her/his information partition Q_1^1 according to the message such as $Q_1^1([e_1(\xi)];\omega) = Q_1^0([e_1(\xi)];\omega) \cap m_P^1([e_1(\xi)];\omega)$. She/he gets her/his revised expected marginal cost $c_1'^1(e_1(\xi);\omega) = f([e_1(\xi)]; Q_1^1([e_1(\xi)];\omega))$, and sends it to the agent 2.

The agent 2 refines his/her information partition Q_2^1 according to the message $[c_1'^1(e_1(\xi);\omega)]$; i.e., $Q_2^2([e_2(\xi)];\omega) = Q_2^1([e_2(\xi)];\omega) \cap m_1^1([e_1(\xi)];\omega)$. He/she revises his/her revised expected marginal cost $c_2'^2(e_2(\xi);\omega) = f([e_2(\xi)]; Q_2^2([e_2(\xi)];\omega))$, and sends $[c_2'^2(e_2(\xi);\omega)]$ as message to the agent 3.

The recipient 3 refines her/his information partition Q_3^2 according to the message $[c_2'^2(e_2(\xi);\omega)]$; i.e., $Q_3^3([e_3(\xi)];\omega) = Q_3^2([e_3(\xi)];\omega) \cap m_2^2([e_2(\xi)];\omega)$. She/he revises his/her revised expected marginal cost $c_3'^3(e_3(\xi);\omega) = f([e_3(\xi)]; Q_3^3(\xi;\omega))$, and sends $[c_3'^3(e_3(\xi);\omega)]$ to the agent 4.

The agent 4 refines his/her information partition Q_4^4 according to the message $[c_3'^3(e_3(\xi);\omega)]$; i.e., $Q_4^4([e_4(\xi)];\omega) = Q_4^3([e_4(\xi)];\omega) \cap m_3^3([e_3(\xi)];\omega)$. He/she revises his/her revised expected marginal cost $c_4'^4(e_4(\xi);\omega) = f(\,[e_4(\xi)]; Q_4^4(\,[e_4(\xi)];\omega))$, and sends $[c_4'^4(e_4(\xi);\omega)]$ as message to the agent 5 and so on.

This protocol is given as

$$\Pr: P \to 1 \to 2 \to 3 \to \cdots \to k \to k+1 \to \cdots \to n \to P \to 1 \to 2 \to \cdots$$

and it is a typical example of cyclic communications. In general,

Definition. The principal-agents model with communication is called *cyclic* if $\Pr$ contains at least one cycle.

We can show that

Theorem 3. *If the system design function* f *satisfies* **DUC***,* **PUD** *and* **CNV** *in the principal-agent model with cyclic communication then there exists a sequence of states* $\{\omega_0, \omega_1, \omega_2, \cdots, \omega_n\} \subset \Omega$ *such that*
$c_P'^\infty(e_k(\xi);\omega_0) = c_1'^\infty(e_k(\xi);\omega_1) = c_2'^\infty(e_k(\xi);\omega_2) = \cdots = c_n'^\infty(e_k(\xi);\omega_n)$ *for* $k \in N$ *and for any* $\xi \in \Omega$*,*

Proof of Theorem 3: The theorem follows immediately from the below proposition in Parikh and Krasucki (1990) by setting $X = [e(\xi)]$

Proposition 2. *Let* $\pi = \langle \Pr, (Q_i^t)_{(i,t)\in \overline{N}\times T}, f \rangle$ *be a communication process with which the common decision function* f *satisfies* **DUC***,* **PUD** *and* **CNV***. Suppose the protocol* $\Pr$ *is cyclic. Then consensus on the limiting values of the decision function about an event* X *can be guaranteed; i.e., there exists a sequence of states* $\{\omega_0, \omega_1, \omega_2, \cdots, \omega_n\} \subset \Omega$ *such that*

$$d_P^\infty(X;\omega_0) = d_1^\infty(X;\omega_1) = d_2^\infty(X;\omega_2) = \cdots = d_n^\infty(X;\omega_n).$$

Proof: Let us first consider the case that $i, j \in \overline{N}$ with $i \to j$. Set $H_i = m_i^\infty(X;\omega)$. It is noted that H_i can be decomposed into the disjoint union of components $Q_i^\infty(X;\xi)$ for $\xi \in H_i$, and that it can be also done into the disjoint union of $Q_j^\infty(X;\xi)$ for $\xi \in H_i$.

Therefore in viewing of **PUD** and **CNV** it can be observed that

$$f(X;H_i) = d_i^\infty(X;\omega)$$

and that there are real numbers $\lambda_1, \lambda_2, \cdots, \lambda_m \in [0,1]$ and states $\xi_1, \xi_2, \cdots, \xi_m \in \Omega$ such that

$$f(X;H_i) = \sum_{k=1}^{m} \lambda_k f(X;Q_i^{\circ}(X;\xi_k)).$$

It follows that for all $\omega \in \Omega$ there is some $\xi_\omega \in Q_i^{\circ}(X;\omega)$ such that

$$f(X;H_i) = d_i^{\circ}(X;\omega) \leq d_i^{\circ}(X;\xi_\omega)$$

Continuing this process according to the *fair* protocol, the below facts can be plainly verified: For each $\omega \in \Omega$ and for any $i \neq j \in \overline{N}$,

$d_i^{\circ}(X;\omega) \leq d_j^{\circ}(X;\xi)$ for some $\xi \in \Omega$;

and

$d_i^{\circ}(X;\omega) \leq d_i^{\circ}(X;\xi) \leq d_i^{\circ}(X;\varsigma) \leq \cdots$ for some $\xi, \varsigma, \cdots \in \Omega$.

Since Ω is finite we can take states $\{\omega_0, \omega_1, \omega_2, \cdots, \omega_n\} \subset \Omega$ such that

$$d_P^{\circ}(X;\omega_0) = d_1^{\circ}(X;\omega_1) = d_2^{\circ}(X;\omega_2) = \cdots = d_n^{\circ}(X;\omega_n),$$

in completing the proof.

7. Concluding Remarks

This article advocates a new approach to treat a moral hazard problem in principal-agent model from epistemic point of view following Bacharach (1985) and Parikh and Krasucki (1990). To highlight the structure of communication between principal and agents is capable of helping us to make progress in `problematic' of classical principal-agent models.

To this end we have proposed the principal-agent model under uncertainty equipped with S5n-knowledge structure. We have seen the communication of expected marginal costs for all resolves the moral hazard under several technical assumptions. The proof of our theorems is based on the technique appeared in Aumann (1976) and Parikh and Krasucki (1996).

It well ends our appraisal on the assumptions on a principal-agent model under uncertainty:

7.1 Common-Prior Assumption

In the principal-agents model under uncertainty we assume for simplicity that a given prior μ is common for all member of $\overline{N}$, and so it is not necessary to assume that the principal and the agents commonly know the prior: In fact, it can be easily seen that if subjective priors $(\mu_i)_{i \in \overline{N}}$ are given instead of a common prior μ then the moral hazard also arises, and further the resolution result is still valid.

7.2 Common System Design Function

In this article we start that there exists a common design function satisfying the above conditions **A1**, **A2** and **A2**^{t} to establish Theorems 2 and 3. Worth noting that these theorems are not true when each of agents and the principal have different system design functions, and the counter example will be easily given.

However it is not yet settled whether we can construct the decision function such that the above conditions **A1**, **A2** and **A2**^{t} are true or whether we can give the further condition under which such the decision function exists. This is a next agenda to our investigation.

7.3 Constant Proportional Rate

If the proportional rate r_k representing k's contribution to a firm depends only on his/her effort for selling productions in the principal-agent model, what solution can we have for the moral hazard problem? This is an interesting problem, and it is also an agenda in future research.

Acknowledgements

The research by T. Matsuhisa was supported by the BUSAIKU-BUHI Foundation for Scientific Researches and it was also supported by Grant-in-Aid for Scientific Research (C) (No.23540178) in the Japan Society for the Promotion of Sciences.

References

Arrow, KJ 1963, 'Uncertainty and Welfare Economics of Medical Care', *American Economic Review* Vol. 53, pp.941-973.

Aumann, R 1976, 'Agreeing to disagree', *Annals of Statistics* Vol.4, pp.1236-1239.

Bacharach, M 1985, 'Some extensions of a claim of Aumann in an axiomatic model of knowledge', *Journal of Economic Theory* Vol.37 pp.167-190.

Holmstrom, B 1982, 'Moral Hazard in Teams', *Bell Journal of Economics* Vol.13, pp.324-340.

Krasuki, P 1993, 'Protocols Forcing Consensus', *Journal of Economic Theory* Vol.70, pp.266-272.

Parikh, R and Krasucki, P 1990, 'Communication, consensus and knowledge', *Journal of Economic Theory* Vol.52, pp.178-189.

Williams, S and Radner, R 1989, 'Efficiency in partnerships when the joint output is uncertain', Discussion Paper No.76, The Center for Mathematical Studies in Economics and Management Science, Kellog School of Management, Northwestern University (Available in http://kellogg.northwestern.edu/research/math/papers/760.pdf)